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INTERFEROMETRIC DETECTION OF OPTICAL PHASE SHIFT USING TWIN BEAMS*

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An interferometric detection scheme to measure optical phase shift with sensitivity beyond the shot noise limit is proposed. The theoretical calculation shows that using the quantum correlated twin beams produced from an optical parametric amplifier as the input fields of a Mach-Zehnder interferometer, the minimum detectable phase shift will exceed the shot noise limit $N^{-1/2}$ and approach the Heisenberg limit N^{-1}. The parametric dependences of the minimum detectable phase shift on the nonlinear interaction, input photon number, and detection efficiency are shown.

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I. INTRODUCTION

The measurement of very small optical phase shifts is of great interest in optics because of the need for sensitive devices such as gravitational wave antennas and laser gyroscopes.[1,2] The Mach-Zehnder (M-Z) interferometer is a good apparatus for detecting phase shifts of light. In a standard M-Z interferometer coherent state light enters from one of its input ports and the other one is 'unused'. Unfortunately, vacuum noise inevitably enters the interferometer from the unused port, therefore the measurement sensitivity of phase shift is limited by the shot noise limit (SNL), which is $\phi_{\rm SNL} = N^{-1/2}$ rad,[3] where N is the photon number of the input coherent state during the measurement interval. In this case, improvement of the interferometer resolution needs very large laser power, which involves buying huge and expensive laser sources. It was found that the SNL can be beaten by injecting quadrature squeezed state light into the empty port of the interferometer whereby the sensitivity can be improved to $\phi = \phi_{\rm SNL}/e^r$, beyond $\phi_{\rm SNL}$,[4,5] where r is the squeezing parameter. An increase in the signal-to-noise ratio (SNR) of 3 dB relative to the SNL was experimentally obtained by Xiao *et al.*[6] in 1989. An alternative method is to drive the interferometer with two Fock states containing the same photon numbers, and it is shown theoretically that the resulting phase sensitivity is limited by the Heisenberg limit,[7] i.e., $\phi = N^{-1}$ which is the best sensitivity of this kind of measurement. Since it is difficult to generate Fock state light, there is no experimental realization published so far.

Parametric twin beams with high quantum correlatin over 9 dB have been produced by several groups around the world.[8,9] The experiments show that twin-beam generation systems, such as nondegenerate parametric amplifiers or optical parametric oscillators

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(OPOs) operated above threshold are robust relative to those producing quadrature squeezed vacuum.[10] Towards practical applications we propose a M-Z interferometer which has both input ports filled respectively by one of two twin beams from an optical parametric amplifier (OPA). The sensitivity of the proposed interferometer is beyond the SNL and approaches the Heisenberg limit.

The paper is organized as follows: the output intensity correlation function is introduced in section II. In Section III the minimum detectable phase shift is deduced. The influence of detector inefficiency on measurement is discussed in section IV.

II. THE OUTPUT INTENSITY CORRELATION FUNCTION

Figure 1 is a diagram of the interferometric detection of phase shift. The twin beams (a_1, a_2) generated by an optical nondegenerate parametric amplifier are injected into the input beamsplitter of the interferometer.

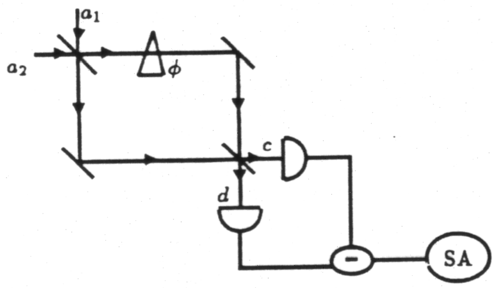

Fig.1. Diagram of interferometer.

A phase shifter of ϕ is placed in one of the arms of the interferometer. The output modes (c, d) from the second beamsplitter are detected by two photodetectors and the photocurrents are subtracted in a 180° power combiner, then the resulting variance of the difference-photocurrent are directed onto the spectrum analyzer where the noise dependence on the phase shift is displayed.

The output modes from the second beamsplitter can be expressed in terms of the input twin beams

$$c = \exp(\mathrm{i}\phi/2)[\cos(\phi/2)a_1 + \mathrm{i}\sin(\phi/2)a_2] \,, \tag{1a}$$

$$d = \exp(\mathrm{i}\phi/2)[\cos(\phi/2)a_2 + \mathrm{i}\sin(\phi/2)a_1] \,, \tag{1b}$$

where the operators a_1, a_2 are related to the input operators of the amplifier by the following formulas:[11]

$$a_1 = \cosh \chi l a_{10} + \sinh \chi l a_{20}^{+} \,, \tag{2a}$$

$$a_2 = \cosh \chi l a_{20} + \sinh \chi l a_{10}^{+} \,. \tag{2b}$$

In Eqs. (2a) and (2b) we have assumed that a_1 and a_2 have the same frequency, amplitude and initial phase but have perpendicular polarization. Here χ is a constant proportional to the second-order nonlinear susceptibility of the crystal, l is the nonlinear interaction length, χl represents the nonlinear parametric gain of the OPA, and a_{10}, a_{20} are the annihilation operators of the input fields of the OPA.

In the case of a_{10} and a_{20} being the vacuum state, the intensity correlation function $(G^{(2)}(\phi, \chi l))$ of the two output fields of the interferometer may be obtained from the following straightforward calculation:

$$\begin{aligned} G^{(2)}(\phi,\chi l) = < c^+cd^+d >= &\cos^4(\phi/2)(\sinh^2\chi l\cosh^2\chi l + \sinh^4\chi l) \\ &+ \sin^4(\phi/2)(\sinh^2\chi l\cosh^2\chi l + \sinh^4\chi l) \\ &+ \sin^2(\phi/2)\cos^2(\phi/2)(\sinh^2\chi l\cosh^2\chi l + \sinh^4\chi l \\ &- 2\sin^2(\phi/2)\cos^2(\phi/2)\sinh^3\chi l\cosh\chi l) \,. \end{aligned} \tag{3}$$

The above expression shows that the correlation depends on the phase shifts and the OPA parameter χl. The dependence of $G^{(2)}$ upon χl is plotted in Fig.2. It is seen that the correlation increases with increasing χl.

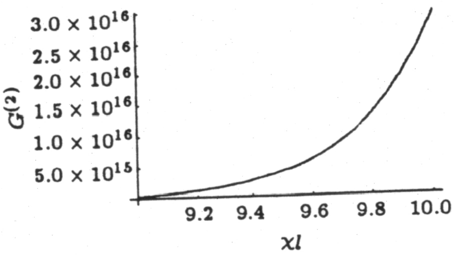

Fig.2. The dependence of intensity correlation on the nonlinear OPA gain.

III. THE MINIMUM DETECTABLE PHASE SHIFT

The phase shift is monitored by measuring the photocurrent difference between the output fields of the interferometer,

$$I_- = c^+c - d^+d \,. \tag{4}$$

Substituting Eq.(1) into Eq.(4) we have

$$I_- = \cos\phi(a_1^+a_1 - a_2^+a_2) - i\sin\phi(a_2^+a_1 - a_1^+a_2) \,. \tag{5}$$

If the initial states of the input signals of the amplifier are coherent states, i.e., $< a_-^+a_1 >=< a_2^+a_2 >= |\alpha|^2$, we obtain

$$< I_- > \doteq 2|\alpha|^2\sin\phi(\cosh\chi l\sinh\chi l\sin 2\theta + 2\cosh^2\chi l\sin\theta) \,, \tag{6}$$

where θ is the initial phase of the coherent states, and can be considered to be zero ($\theta = 0$).

The variance of the photocurrent difference may be expressed as follows:

$$\begin{aligned} V < I_- > = < (I_-)^2 > - < (I_-) >^2 = &2|\alpha^2|\cos^2\phi + \sin\phi\cos\phi|\alpha^2|A \\ &+ \sin^2\phi\Big[|\alpha|^4B - |\alpha|^4C + |\alpha|^2D + E\Big] \,, \end{aligned} \tag{7}$$

where

$$
\begin{aligned}
A =&4\cosh\chi l \sinh\chi l \sin 2\theta \,, \\
B =&2(\cosh^4\chi l + \sinh^4\chi l + 3\cosh^2\chi l \sinh^2\chi l) + 4(\cosh^3\chi l \sinh\chi l \\
&+ \cosh\chi l \sinh^3\chi l)\cos\theta - 2(\cosh^4\chi l + \sinh^4\chi l)\cos 2\theta \\
&- 4(\cosh^3\chi l \sinh\chi l + \cosh\chi l \sinh^3\chi l)\cos 3\theta - 2\cosh^2\chi l \sinh^2\chi l \cos 4\theta \,, \\
C =&4[\cosh\chi l \sinh\chi l \sin 2\theta + (2\cosh^2\chi l - 1)\sin\theta]^2 \,, \\
D =&8[\cosh^3\chi l \sinh\chi l + \cosh\chi l \sinh^3\chi l \cos\theta \\
&+ 2(\cosh^4\chi l + \sinh^4\chi l + 6\cosh^2\chi l \sinh^2\chi l)] \,, \\
E =&4\cosh^2\chi l \sinh\chi l \,.
\end{aligned}
$$

The signal-to-noise ratio is defined as

$$\mathrm{SNR} = \frac{< I_- >}{\sqrt{V(I_-)}} \,. \tag{8}$$

For an efficient measurement beyond the shot noise limit the SNR should be larger than 1, this criterion is expressed by the following inequality in terms of ϕ:

$$(2C|\alpha|^4 - B|\alpha|^4 - D|\alpha|^2 - E)\sin^2\phi - A|\alpha|^2\cos\phi\sin\phi - 2|\alpha^2|\cos^2\phi \geq 0 \,. \tag{9}$$

In practice, the phase shift is so small that we can set $\sin\phi = \phi$ and $\cos\phi = 1$. If $|\alpha|^2 = n$ is the average photon number of the initial field injected into the OPA, Eq.(9) becomes

$$[2n^2C - n^2B - nD - E]\phi^2 - nA\phi - 2n \geq 0 \,. \tag{10}$$

Since $2n^2C - n^2B - nD - E > 0$, the solution of Eq.(10) is

$$\phi \geq \frac{A + \sqrt{A^2 + 8[(2C - B)n - D - E/n]}}{2(2C - B)n - D - E/n} \,. \tag{11}$$

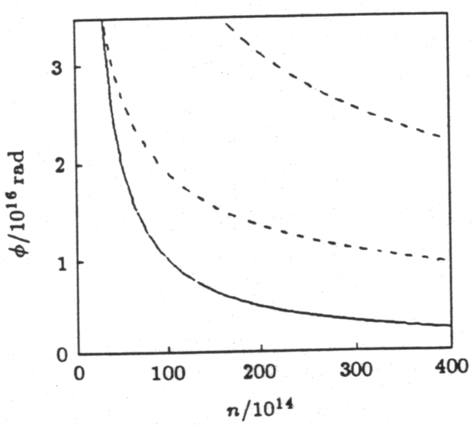

Fig.3. The minimum phase shift vs the number of incident photons. The solid line is for the Heisenberg limit, dashed line for χl=9.2 and dot-dashed line for χl=9.1.

The minimum detectable phase shift $\phi_{\min}$ is obtained by taking the equal sign in Eq.(11). The dependence of $\phi_{\min}$ on photon number n is plotted in Fig.3. The solid line is the Heisenberg limit ($\phi = 1/N$), the dashed line and dot-dashed line are given by Eq.(11), taking respectively $\chi l = 9.2$ and χl=9.1. As expected, for larger χl the minimum phase shift is closer to the Heisenberg limit. This means that the measurement sensitivity is increased as the correlation between a_1 and a_2 is improved by a stronger nonlinear interaction in the OPA.

IV. ANALYSIS OF DETECTION INEFFICIENCY

It is known that inefficient photodetectors can introduce noise into any optimal measurement scheme. A detector with quantum efficiency η is equivalent to a beamsplitter which mixes the input mode (a) with a vacuum mode (v), followed by detecting the output mode with a perfect efficient detector.[12]

For brevity, we assume that the photodetecteors D1 and D2 have the same quantum efficienes, i.e., $\eta_1 = \eta_2 = \eta$, then the annihilation operators for the modes detected by D1 and D2 are given by

$$j = \eta^{1/2}c + (1-\eta)^{1/2}v \,, \tag{12a}$$

$$k = \eta^{1/2}d + (1-\eta)^{1/2}v \,, \tag{12b}$$

where v is the annihilation operator for the vacuum state.

The analyzed photocurrent difference and its variance are, respectively,

$$< j^+j - k^+k > = \eta < c^+c - d^+d > \tag{13}$$

and

$$\begin{aligned}[\Delta(j^+j - k^+k)]^2 =& \eta^2 < (c^+c - d^+d)^2 > - \eta^2 < c^+c - d^+d >^2 \\ &+ \eta(\eta-1)[< c^+c > + < d^+d >] \,.\end{aligned} \tag{14}$$

From the above, Eq.(14) becomes

$$\begin{aligned}[\Delta(j^+j - k^+k)]^2 =& \eta^2[n^2(B-C) + nD + E]\sin^2\phi + \eta^2 nA \sin\phi\cos\phi \\ &+ \eta(1-\eta)nF\sin\phi + 2\eta^2 n\cos^2\phi + \eta(1-\eta)nM + \eta(1-\eta)N \,,\end{aligned} \tag{15}$$

where

$$\begin{aligned}F &= 2[(2\cosh^2\chi l - 1)\sin\theta + \cosh\chi l \sinh\chi l \sin 2\theta] \,, \\ M &= 4[(2\cosh^2\chi l - 1) + 2\cosh\chi l \sinh\chi l \cos\theta] \,, \\ N &= 4\sinh\chi l \,.\end{aligned}$$

The expression for the SNR for inefficient detection is

$$\mathrm{SNR} = \frac{< j^+j - k^+k >}{\sqrt{[\Delta(j^+j - k^+k)]^2}} \geq 1 \,. \tag{16}$$

From Eqs.(15) and (16) we get the inequality

$$A'\phi^2 - B'\phi - C' \geq 0 \,, \tag{17}$$

where

$$\begin{aligned}A' &= (2C - B)n - D - E/n \\ B' &= A + F\eta(1-\eta)/\eta^2 \,, \\ C' &= M\eta(1-\eta)/\eta^2 + N\eta(1-\eta)/\eta^2 n + 2 \,.\end{aligned}$$

The solution of Eq.(17) is thus

$$\phi \geq (B' + \sqrt{B'^2 + 4A'C'})/2A' . \tag{18}$$

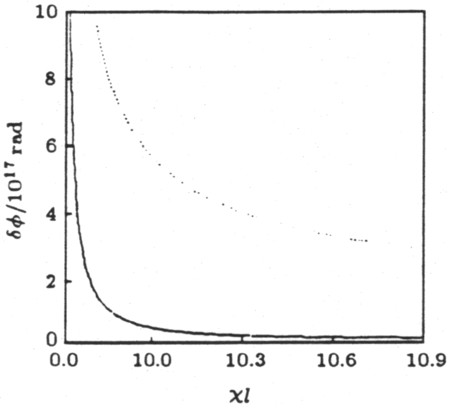

Fig.4. The minimum phase shift vs the amplifier gain (χl) for $n = 2 \times 10^{18}$. Solid line: $\eta = 1$; Dotted line: $\eta = 0.99$.

The minimum phase shift as a function of nonlinear interaction gain (χl) is shown in Fig.4 for different efficiencies η=1 (solid line) and η = 0.99 (dashed line). The influence of detection inefficiency on sensitivity is significant. The physical reason is that inefficient detection destroys the intensity correlation between the twin beams, which is the key to improving the sensitivity in the proposed device.

V. CONCLUSION

We have shown that the Heisenberg limit can be approached in the detection of phase shift by using twin beams as the input fields of a M-Z interferometer. The dependence of minimum detectable phase shift on the nonlinear gain of the parametric amplifier and the detector quantum efficiency is shown.

The sensitivity of the interferometer is directly related to the quantum correlation between the twin beams, which is determined by the nonlinear optical gain of the OPA. Thus the larger the gain is, the higher the sensitivity is.

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